Academic Year of 2020
Admission to the Master's Program
Department of Intelligence Science and Technology
Graduate School of Informatics, Kyoto University
(Fundamentals of Informatics)
(International Course)

13:00 - 15:00, February 5, 2020

NOTES

1. This is the Question Booklet in 5 pages including this front cover.
2. Do not open the booklet until you are instructed to start.
3. After start, check the number of pages and notify proctors (professors) immediately if you find missing pages or unclear printings.
4. This booklet has 4 questions written in English. **Solve all questions.**
5. Write your answers in English, unless specified otherwise.
6. Read carefully the notes on the Answer Sheets as well.

Use one answer sheet for each of F1–1, F1–2, F2–1, and F2–2.

Q. Let a and b be real numbers. We define a matrix

$$\mathbf{A} = \begin{pmatrix} 0 & a & b \\ a & 0 & 1 \\ b & 1 & 0 \end{pmatrix}.$$

In all of the questions (1), (2), (3), and (4), assume that an eigenvalue (characteristic value) of $\mathbf{A}$ is 1.

(1) What is the relation between a and b?

(2) When $a = 0$, list all eigenvalues of $\mathbf{A}$.

In questions (3) and (4), let us additionally assume that $a > 0$ and also that the characteristic polynomial of $\mathbf{A}$ has a double (multiple) root of an integer.

(3) Give the eigenvector $\mathbf{v}$ corresponding to the smallest eigenvalue of $\mathbf{A}$.

(4) Give a pair of vectors $\mathbf{u}$ and $\mathbf{w}$ satisfying all of the following conditions:

- both $\mathbf{u}$ and $\mathbf{w}$ are eigenvectors corresponding to the eigenvalue which is the double root of the characteristic polynomial of $\mathbf{A}$,
- $\mathbf{u}$ is orthogonal to both $\mathbf{e} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{w}$, and
- $||\mathbf{u}|| = ||\mathbf{w}|| = 1$.

Use one answer sheet for each of F1–1, F1–2, F2–1, and F2–2.

Q. Answer the following questions. In the following, n is a positive integer and $\log x$ denotes $\log_e x$.

(1) Let $f(x) = e^x(x^2 + x)$. Derive $\dfrac{d^n f(x)}{dx^n}$.

(2) (i) Find the Maclaurin series of $\log(1 - x)$.

(ii) Derive the following limit:

$$\lim_{x \to 0} \frac{x + \log(1 - x)}{x^2}.$$

(3) The Maclaurin series of e^x is given by $e^x = 1 + x + \dfrac{x^2}{2!} + \cdots + \dfrac{x^n}{n!} + \cdots$.

Derive the following limit:

$$\lim_{x \to 0} \frac{x \sin x}{e^{x^2} - \cos x}.$$

Use one answer sheet for each of F1–1, F1–2, F2–1, and F2–2.

Q. Consider an array A of integers. Answer the following questions.

(1) When A is [18, 61, 8, 23, 45, 37, 97], draw a heap for this array with the maximum integer at its root.

(2) Explain the procedure of heapsort for the array A.

(3) Explain the procedure of quicksort for the array A.

(4) Discuss the worst-case and average-case time complexities of heapsort and quicksort.

Use one answer sheet for each of F1–1, F1–2, F2–1, and F2–2.

Q.1 In the following, x_i takes either 0 or 1. Symbols $\wedge$, $\vee$, and $\overline{x}$ represent the logical conjunction, the logical disjunction, the negation of x, respectively. Let n be an arbitrary positive integer. Answer (1), (2), and (3).

(1) List all possible combinations of (x_1, x_2) that satisfy $(x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2) = 1$.

(2) List all possible combinations of $(x_1, \ldots, x_n)$ that satisfy

$$(x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2}) \wedge (x_2 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3}) \wedge \ldots$$
$$\wedge (x_{n-1} \vee x_n) \wedge (\overline{x_{n-1}} \vee \overline{x_n}) \wedge (x_n \vee x_1) \wedge (\overline{x_n} \vee \overline{x_1}) = 1.$$

(3) List all possible combinations of $(x_1, x_2, x_3, x_4, x_5)$ that satisfy

$$\overline{(\overline{x_1} \vee (\overline{x_3} \wedge \overline{x_4}) \vee (x_1 \wedge x_4) \vee (x_1 \wedge \overline{x_2} \wedge x_3))}$$
$$\wedge (\overline{(\overline{x_4} \wedge \overline{x_5}) \vee (\overline{x_1} \wedge \overline{x_4} \wedge x_5)) \vee (x_1 \wedge \overline{x_2}) \vee x_4}) = 1.$$

Q.2 Suppose that an algorithm A solves a problem P in $O(f_n)$ time, where an integer n (≥ 3) is the size of P. If $f_n = f_{n-1} + f_{n-2}$, $f_2 = 1$ and $f_1 = 1$ hold, the time complexity of A takes the form of $O(a^n)$. Answer the minimum a with 3 digits.