# Academic Year of 2019
# Admission to the Master's Program
# Department of Intelligence Science and Technology
# Graduate School of Informatics, Kyoto University
# (Fundamentals of Informatics)
## (International Course)

13:30 - 15:00, February 6, 2019

**NOTES**

1. This is the Question Booklet in 3 pages including this front cover.
2. Do not open the booklet until you are instructed to start.
3. After start, check the number of pages and notify proctors (professors) immediately if you find missing pages or unclear printings.
4. This booklet has 2 questions written in English. **Solve all questions.**
5. Write your answers in English, unless specified otherwise.
6. Read carefully the notes on the Answer Sheets as well.

**Q. 1** Answer the following questions.

1.1 Given two sequences of length $n$, write pseudocode of an $O(n^2)$-time algorithm to find a longest common subsequence. Note that a subsequence does not necessarily have to be contiguous. For example, if $X = (A, B, C, B, D, A, B)$ and $Y = (B, D, C, A, B, A, B)$, the sequence $(B, C, A)$ is a common (but not longest) subsequence of $X$ and $Y$.

1.2 Given a sequence of permuted integers from 1 to $n$, write pseudocode of an $O(n \log n)$-time algorithm to find the length of the longest monotonically increasing subsequence.

**Q. 2** Give context-free grammars that generate the following languages. In all parts, the alphabet (terminal character set) is $\{0, 1\}$.

2.1 $\{ \omega \mid \omega$ contains at least three 1s $\}$

2.2 $\{ \omega \mid$ the length of $\omega$ is odd and its middle symbol is a 0 $\}$

2.3 $\{ \omega \mid \omega = \omega^{\mathcal{R}}$, where $\mathcal{R}$ denotes writing backwards, that is, $\omega$ is a palindrome $\}$

**Q. 1** In a game of two players, Alice is trying to guess a secret 4-bit number chosen by Bob. Alice gets information about the secret number by proposing a 4-bit number. Bob replies by indicating how many **1**s in the proposed number are in the same position as in the secret number. For example, if Bob chose the 4-bit secret number **1110** and Alice proposes **1100**, she will get the reply "*two 1s in correct position*" from Bob. If Alice then proposes **0100**, she will get the reply "*one 1 in correct position.*" And for **1110**, she will get the reply "*three 1s in correct position.*"

1.1 When Alice proposes the number **n**, let us denote with $X_{\mathbf{n}}$ the number of **1**s in the same position as in the secret number. If we assume that Bob chooses the secret number according to a uniform distribution, then each $X_{\mathbf{n}}$ can be seen as a random variable and we can compute its entropy. The entropy of $X_{1111}$ is H($X_{1111}$)$\approx$ 2.025. Compute the entropy of $X_{0000}$, $X_{1000}$, $X_{1100}$ and $X_{1110}$. (You can use the approximation $\frac{3}{4}log_2 3 \approx 1.19$.)

1.2 What is the mutual information of $X_{1010}$ and $X_{0101}$? What is the conditional entropy of $X_{1100}$ given $X_{1000}$? What is the mutual information of $X_{1000}$ and $X_{1100}$?

1.3 Using the result of question 1.1, prove that any strategy for finding Bob's number correctly will require Alice to propose at least three numbers in the worst case.

1.4 It is actually possible for Alice to always correctly guess Bob's number using at most three propositions. One possible strategy is to always propose the numbers **1110**, **1001** and **0011**. If, for a given secret number, we have $X_{1110} = 1$, $X_{1001} = 0$ and $X_{0011} = 1$, what is the secret number?

1.5 According to question 1.1, proposing **1111** gives the most information on average to Alice (i.e. H($X_{1111}$) has the highest value). However it can be proven that no optimal strategy for Alice involve proposing **1111** (i.e. if Alice proposes **1111**, she will still need to make three additional propositions to find the correct number in the worse case). Explain how this seemingly paradoxical situation can happen, in terms of Information Theory and Mutual Information?

**Q. 2** Minister X claims that his policies for preventing motorbike accidents caused by young drivers are successful: the number of young drivers causing motorbike accidents has halved. His political opponent, Mr. Y, argues on the other hand that the proportion of motorbike accidents that are caused by young drivers has increased from 20% to 40%.

2.1 Explain briefly why Minister X's and Mr. Y's assertions do not contradict with each other.

2.2 $A_0$ is the initial total number of accidents (with both young and old drivers), and $A_1$ is the total number of accidents after Minister X's policies. Express $A_1$ in terms of $A_0$.