Academic Year of 2017
Admission to the Master's Program
Department of Intelligence Science and Technology
Graduate School of Informatics, Kyoto University
(Fundamentals of Informatics)
(International Course)

13:30 - 15:00, February 9, 2017

NOTES

1. This is the Question Booklet in 3 pages including this front cover.
2. Do not open the booklet until you are instructed to start.
3. After start, check the number of pages and notify proctors (professors) immediately if you find missing pages or unclear printings.
4. This booklet has 2 questions written in English. **Solve all questions.**
5. Write your answers in English, unless specified otherwise.
6. Read carefully the notes on the Answer Sheets as well.

Master's Program	Fundamentals of Informatics	Question Number	F-1

Q. 1 Binary search trees are a widely used data structure in computer science.

1.1 Give the definition of a binary search tree in 100 words.
1.2 When we want to search for the number 363 in a binary search tree that has distinct integers in the range of [1, 1000], which of the following sequences could NOT be the sequence of nodes visited? Give the reason of your choice(s).

(a) 925, 202, 911, 240, 912, 245, 363.
(b) 948, 218, 911, 237, 888, 258, 362, 363.
(c) 7, 249, 401, 395, 330, 344, 394, 363.
(d) 935, 278, 347, 621, 299, 392, 358, 363.
(e) 12, 399, 387, 219, 266, 380, 379, 278, 363.

1.3 Suppose that the search for a key k in a binary search tree ends up in a leaf. Consider three sets: A, the keys to the left of the search path; B, the keys on the search path; and C, the keys to the right of the search path. Can we claim that any three keys $a \in A, b \in B$, and $c \in C$ must satisfy $a \leq b \leq c$? If yes, give your proof. If no, give a possible counterexample that has the minimum number of nodes, and explain why it is a counterexample.

Q. 2 The problem of finding the pth smallest element in an array Q of n distinct numbers is named **Selection**.

2.1 Give a **Selection** algorithm that takes $\Theta(n\log n)$ time for an unsorted array Q.
2.2 Consider another **Selection** algorithm using partitioning of Q that works in linear time on average. Give a pseudo-code of the algorithm and a proof of its time complexity.

Q.1 Answer the following questions on the 4-variable logic function

$$f(x_1,x_2,x_3,x_4) = (x_1 + (x_2x_3')')x_4' + (x_1x_3)'x_2x_4,$$

where x_i' denotes the logical negation of x_i.

1. Show a Karnaugh map (K-map) of $f(x_1,x_2,x_3,x_4)$.
2. Show all the prime implicants of $f(x_1,x_2,x_3,x_4)$.
3. Show all the minimum sum-of-products expressions of $f(x_1,x_2,x_3,x_4)$.

Q.2 Answer the following questions related to a Mealy-type sequential machine M that detects patterns `0011` and `1011` in a binary signal sequence. M produces `1` at the time when the last signal of the patterns to be detected comes, and produces `0` at the other time. For example, when M is fed `011011101101`, it produces `000001000100`.

1. Show the state transition table of M. The number of states must be minimized.
2. We would like to design a synchronous sequential circuit that realizes M with a minimum number of D flip-flops. Let X denote the input of the sequential circuit, and Q_i and Q_i' denote the outputs of the ith D flip-flop ($i = 1,2,\ldots$). Using X, X', Q_i and Q_i', where X' and Q_i' are the logical negations of X and Q_i, give the excitation functions D_1, D_2, ... of the D flip-flops and the output function Z of the sequential circuit in the minimum sum-of-products form. In the answer, show the state assignment clearly.

   The D flip-flop has an input and two outputs, and captures the value of the input. The captured value and its negation become the outputs at the next clock. The outputs of all D flip-flops are reset to `0` before starting to input a binary signal sequence.