Academic Year of 2016
Admission to the Master's Program
Department of Intelligence Science and Technology
Graduate School of Informatics, Kyoto University
(Fundamentals of Informatics)
(International Course)

13:30 - 15:00, February 9, 2016

NOTES

1. This is the Question Booklet in 3 pages including this front cover.
2. Do not open the booklet until you are instructed to start.
3. After start, check the number of pages and notify proctors (professors) immediately if you find missing pages or unclear printings.
4. This booklet has 2 questions written in English. **Solve all questions.**
5. Write your answers in English, unless specified otherwise.
6. Read carefully the notes on the Answer Sheets as well.

Q. 1 Stack, queue, and tree are three widely used data structures in computer science, which store data so that it can be used efficiently in a computer.

1.1 Describe the basic principle and operations of a queue.
1.2 Discuss how to implement a queue using stack(s).
1.3 Breadth-first search (BFS) is an algorithm for traversing or searching a tree. Given a tree T and a starting vertex v of T, provide a non-recursive implementation of BFS that uses a queue to search the entire tree T.

Q. 2 Divide and conquer (D&C) is an algorithm design paradigm that is used to efficiently solve a variety of problems, such as sorting (e.g. quicksort and merge sort), searching, multiplying large numbers, and so on.

2.1 Describe the basic principle of D&C.
2.2 Given a one-dimensional array A of $n > 1$ non-zero integers, describe a D&C algorithm that finds a contiguous sub-array in A which has the maximum sum, and discuss the time complexity of your algorithm.

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Q. We consider binary strings $\mathbf{x} = [x_1, \ldots, x_n] \in \{0,1\}^n$ that contain $n \geq 1$ bits. To communicate such binary strings to a receiver, the sender can only use a noisy communication channel. The channel is noisy because it flips each bit of an input string $\mathbf{x}$ independently at random with probability $f < 1/2$, to output a random string $N(\mathbf{x})$ of the same size n.

1. For a binary string $\mathbf{y} = [y_1, \ldots, y_n] \in \{0,1\}^n$, give the probability $P(N(\mathbf{x}) = \mathbf{y})$. Use the Hamming distance $d(\mathbf{x}, \mathbf{y})$ defined below to write that formula.

$$d(\mathbf{x}, \mathbf{y}) \stackrel{\text{def.}}{=} \sum_{i=1}^{n} \delta_i, \text{ where for } 1 \leq i \leq n, \ \delta_i \stackrel{\text{def.}}{=} \begin{cases} 1 \text{ if } x_i \neq y_i, \\ 0 \text{ otherwise.} \end{cases}$$

To handle noise, the sender and receiver can agree on using a code (C, D), that is, an encoding function $C : \{0,1\}^n \to \{0,1\}^m$ and a decoding function $D : \{0,1\}^m \to \{0,1\}^n$, where $m \geq n$. The string $\mathbf{x}$ is first encoded as $C(\mathbf{x})$ by the sender; it is then sent and corrupted with random noise as it goes through the channel to form a random string $N(C(\mathbf{x}))$; it is then decoded by the receiver as $D(N(C(\mathbf{x})))$. The probability of error per bit for the code (C, D) and the input string $\mathbf{x}$ is defined as:

$$\rho_{C,D}(\mathbf{x}) \stackrel{\text{def.}}{=} [P(\mathbf{x} \neq D(N(C(\mathbf{x}))))]^{1/n}.$$

For $r \geq 1$, the repetition code (C_r, D_r) is such that $m = rn$; given $\mathbf{x}$, the encoding function considers each bit of $\mathbf{x}$ and repeats it r times, to output:

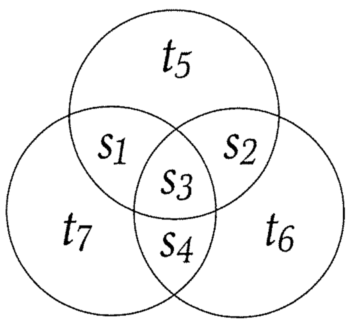
$$C_r(\mathbf{x}) = [\underbrace{x_1, x_1, \cdots, x_1}_{r \text{ times}}, \underbrace{x_2, x_2, \cdots, x_2}_{r \text{ times}}, \ldots, \underbrace{x_n, x_n, \cdots, x_n}_{r \text{ times}}].$$

2. Suppose $r = 3$. Describe the majority-vote decoding function D_3 that can be used with C_3. Give $\rho_{C_3,D_3}(\mathbf{x})$, showing it is the same for all $\mathbf{x}$.

3. Assuming r is odd and using the majority-vote decoding function D_r, give $\rho_{C_r,D_r}(\mathbf{x})$.

A 4/7 Hamming code encodes a string of $n = 4$ bits into $m = 7$ bits using parity checks.

4. Define the Hamming encoding function C_H which agrees with the figure below, and explain how a string $\mathbf{s}$ of four bits can be encoded as a string of seven bits using C_H. What is $C_H([1,1,0,1])$?



5. Using the modulo-2 arithmetic $(0 + 1 = 1 + 0 = 1; 1 + 1 = 0 + 0 = 0)$, write the encoding $C_H(\mathbf{s})$ of a string $\mathbf{s}$ of four bits as the product of $\mathbf{s}$, seen as a row vector, and a 4×7 matrix $\mathbf{G}$ that you will provide.

6. Describe an efficient decoding function D_H that can be used with C_H.